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SAMPLE PAPER TEST 08 (STANDARD) (2019-20) (SAMPLE ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**

CLASS : X **DURATION : 3 HRS**

General Instruction:

(i) All the questions are compulsory.

(ii) The question paper consists of 40 questions divided into 4 sections A, B, C, and D.

(iii) **Section A** comprises of 20 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 8 questions of **3 marks** each. **Section D** comprises of 6 questions of **4 marks** each.

(iv) There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600 (c) 2400 (d) 3600

Ans: (d) 3600

2. The decimal expansion of the rational number $\frac{33}{2^2.5}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) more than 3 decimal places

Ans: (b) two decimal places

3. If sec 5A = cosec(A + 30^0), where 5A is an acute angle, then the value of A is

(a) 15° (b) 5° (c) 20° (d) 10°

Ans: (d) 10°

4. The points (7, 2) and (–1, 0) lie on a line

(a) 7y = 3x – 7 (b) 4y = x + 1 (c) y = 7x + 7 (d) x = 4y + 1

Ans: (b) 4y = x + 1

5. If $\sin\theta = \frac{1}{3}$, then find the value of $(2\cot^2\theta + 2)$

(a) 9 (b) 11 (c) 18 (d) none of these

Ans: (c) 18

6. If $\cos\theta = \frac{1}{2}$, $\sin\phi = \frac{1}{2}$ then find the value of $\theta + \phi$.

(a) 0° (b) 90° (c) 60° (d) 30°

Ans: (b) 90°

7. If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3), then the value of x is

(a) 0 (b) 1 (c) 2 (d) 3

Ans: (c) 2

8. Ratio in which the line 3x + 4y = 7 divides the line segment joining the points (1, 2) and (– 2, 1) is

(a) 3 : 5 (b) 4 : 6 (c) 4 : 9 (d) None of these

Ans: (c) 4 : 9

9. The point on the x -axis which if equidistant from the points A(– 2, 3) and B(5, 4) is
(a) (0, 2) (b) (2, 0) (c) (3, 0) (d) (– 2, 0)
Ans: (b) (2, 0)

10. Consider the following frequency distribution :

Class	Frequency
0-5	13
6-11	10
12-17	15
18-23	8
24-29	11

The upper limit of the median class is
(a) 17 (b) 17.5 (c) 18 (d) 18.5
Ans: (b) 17.5

11. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by _____
Ans: 700%

12. The value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are _____
Ans: 0 and 8

OR

If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is _____
Ans: 4

13. In triangles ABC and DEF, $\angle A = \angle E = 40^0$, AB : ED = AC : EF and $\angle F = 65^0$, then $\angle B = \underline{\mathbf{75^0}}$

14. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is _____
Ans: 51

15. The value of y if the first three terms of an AP respectively are 3y – 1, 3y + 5 and 5y + 1 is _____
Ans: 5

16. If LCM (480, 672) = 3360, find HCF (480,672).
Ans: HCF = 96

17. If radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.
Ans: 6 cm

OR

If angle between two radii of a circle is 130°, find the angle between the tangents at the ends of the radii.
Ans: 50^0.

18. If α, β are the roots of the quadratic equation $4x^2 + 3x + 7 = 0$, then find $\frac{1}{\alpha} + \frac{1}{\beta}$

Ans: $\frac{-3}{7}$

19. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{9}{4}$, and BC = 15 cm, then find PR.

Ans:

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta QRP)} = \left(\frac{BC}{RP}\right)^2 \Rightarrow \frac{9}{4} = \left(\frac{15}{PR}\right)^2 \Rightarrow PR = 10\,\text{cm}$$

20. Which term of the AP: 21, 42, 63, 84,... is 210?

Ans: 10th term

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. A box contains cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that number on the drawn card is (i) a prime number (ii) a number divisible by 3

Ans: $(i)\frac{8}{20} = \frac{2}{5}$ $(ii)\frac{6}{20} = \frac{3}{10}$

OR

The King, Queen and Jack of clubs are removed from a pack of 52 cards and then the remaining cards are well shuffled. A card is selected from the remaining cards. Find the probability of getting a card (i) of spade (ii) of black king

Ans: $(i)\frac{13}{49}$ $(ii)\frac{1}{49}$

22. In an A.P if sum of its first n terms is $3n^2 + 5n$ and its kth term is 164, find the value of k.

Ans:

$S_1 = 3.1^2 + 5.1 = 8 = a_1$

$S_2 = 3.2^2 + 5.2 = 22 = a_1 + a_2$

$a_2 = 22 - 8 = 14 \Rightarrow d = 6$

$t_k = 164 \Rightarrow 8 + (k-1)6 = 164 \Rightarrow k = 27$

23. Ramesh has a wooden piece cube whose edge is 9 cm. He wants to make a toy using a right circular cone. He asks his friend Mohan about the largest right circular cone that can be cut out from wooden cube. Mohan suggested the method to find it. Find the volume of the largest right circular cone that can be cut out of a cube? [Use = 22/7]

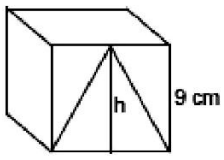
Ans:

Radius of cone $= \frac{1}{2} \times$ edge of cube $= \frac{9}{2}$ cm

Height of cone, $h = 9$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 9$

$= 190.928\ \text{cm}^3 = 190.93\ \text{cm}^3$



24. X is a point on the side BC of ΔABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that $TX^2 = TB \times TC$

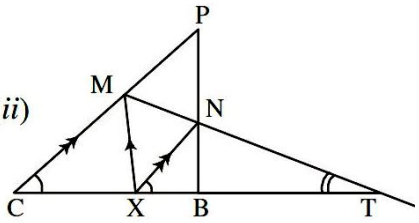
Ans:

$\Delta TXN \sim \Delta TCM$

$\Rightarrow \quad \frac{TX}{TC} = \frac{XN}{CM} = \frac{TN}{TM} \quad \Rightarrow TX \times TM = TC \times TN \ ....(i)$

Again, $\Delta TBN \sim \Delta TXM$

$\Rightarrow \quad \frac{TB}{TX} = \frac{BN}{XM} = \frac{TN}{TM} \quad \Rightarrow \quad TM = \frac{TN \times TX}{TB} \ ...(ii)$



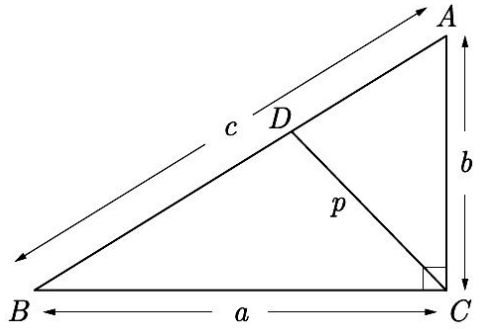
using (ii) in (i), we get

$TX^2 \times \frac{TN}{TB} = TC \times TN \quad \Rightarrow \quad TX^2 = TC \times TB$

OR

ABC is a right triangle right angled at C. Let BC = a, CA = b, AB = c and p be the length of perpendicular from C to AB. Prove that cp = ab.

Ans:



In the given figure CD = AB, and CD = p

Area ΔABC = $\frac{1}{2}$ x base x height = $\frac{1}{2}$ x AB x CD = $\frac{1}{2}$ x cp

Also, Area of ΔABC = $\frac{1}{2}$ x BC x AC = $\frac{1}{2}$ x ab

Thus $\frac{1}{2}$ x cp = $\frac{1}{2}$ ab

$\Rightarrow$cp = ab

25. Aditya placed a ladder against a wall to draw some painting on the wall. Later he observed that the length of the ladder is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal. When Aditya measures the distance of the foot of the ladder from the wall is 4m, find the length of the ladder.

Ans: 60^0 and length of the ladder = 8m

26. In the below figure, ABC is a triangle in which $\angle$B = 90º, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find radius r of in-circle.

Ans:

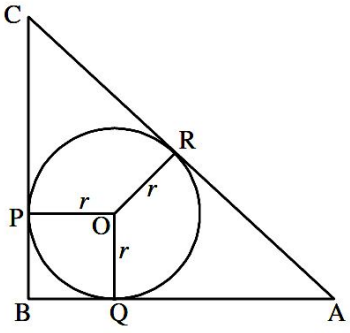
$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{14^2 + 48^2} = \sqrt{2500} = 50\,cm$

$\angle OQB = 90° \Rightarrow OPBQ$ is a square

$\Rightarrow \quad BQ = r, QA = 14 - r = AR$

Again $PB = r, \quad PC = 48 - r \Rightarrow RC = 48 - r$



SECTION – C

Questions 27 to 34 carry 3 marks each.

27. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Ans: NCERT Exercise 1.1 Q2

OR

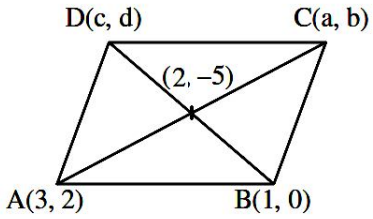
Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Ans: NCERT Exercise 1.1 Q5

28. Four friends A, B, C and D are playing in the park. They took one long rope and try to form a parallelogram. The position of A is (3, 2) and that of B is (1, 0). They took two more rope to form two diagonals and the ropes are intersecting at (2, –5). Now find coordinates of the other two friends C and D.

Ans:

Let the coordinates of C and D be (a, b) and (c, d)



$\therefore \dfrac{3+a}{2} = 2 \Rightarrow a = 1$ and $\dfrac{2+b}{2} = -5 \Rightarrow b = -12$

Also $\dfrac{c+1}{2} = -5 \Rightarrow c = 3$

and $\dfrac{d+0}{2} = -5 \Rightarrow d = -10$

Coordinate of C and D are (1, –12) and (3, –10)

29. Obtain all zeroes of $3x^4 - 15x^3 + 13x^2 + 25x - 30$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans:

$p(x) = 3x^4 - 15x^3 + 13x + 25x - 30$

$x - \sqrt{\frac{5}{3}}$ and $x + \sqrt{\frac{5}{3}}$ are factors of p(x)

$\Rightarrow \quad x^2 - \frac{5}{3}$ or $\frac{(3x^2 - 5)}{3}$ is a factor of p(x)

$p(x) = \frac{(3x^2 - 5)}{3}(x^2 - 5x + 6) = \frac{1}{3}(3x^2 - 5)(x - 3)(x - 2)$

$\therefore$ Zeroes of p(x) are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, 2$ and 3

30. Prove that $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

(NCERT Introduction to Trigonometry Exercise – 8.4, Q5 (x))

OR

Evaluate: $\dfrac{\cos 58^0}{\sin 32^0} + \dfrac{\sin 22^0}{\cos 68^0} - \dfrac{\cos 38^0 \cos ec 52^0}{\sqrt{3}\left(\tan 18^0 \tan 35^0 \tan 60^0 \tan 72^0 \tan 55^0\right)}$

Ans:

$$\frac{\cos 58^\circ}{\sin 32^\circ}+\frac{\sin 22^\circ}{\cos 68^\circ}-\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\sqrt{3}(\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ)}$$

$$=\left(\frac{\cos 58^\circ}{\sin (90-58^\circ)}+\frac{\sin 22^\circ}{\cos (90-22^\circ)}\right)-\frac{\cos 38^\circ \operatorname{cosec} (90-38)^\circ}{\sqrt{3}(\tan 18^\circ \tan 35^\circ \cdot \sqrt{3} \cdot \cot 18^\circ \cot 35^\circ)}$$

$$=1+1-\frac{\cos 38^\circ \sec 38^\circ}{3.1} \quad =2-\frac{1}{3}=\frac{5}{3}$$

31. For what values of m and n the following system of linear equations has infinitely many solutions. $3x + 4y = 12$ and $(m + n)x + 2(m – n)y = 5m – 1$

Ans:

For infinitely many solutions.

$$\frac{3}{m+n}=\frac{4}{2(m-n)}=\frac{-12}{-(5m-1)}$$

$$\frac{3}{m+n}=\frac{4}{2(m-n)} \Rightarrow m-5n=0 \qquad ...(1)$$

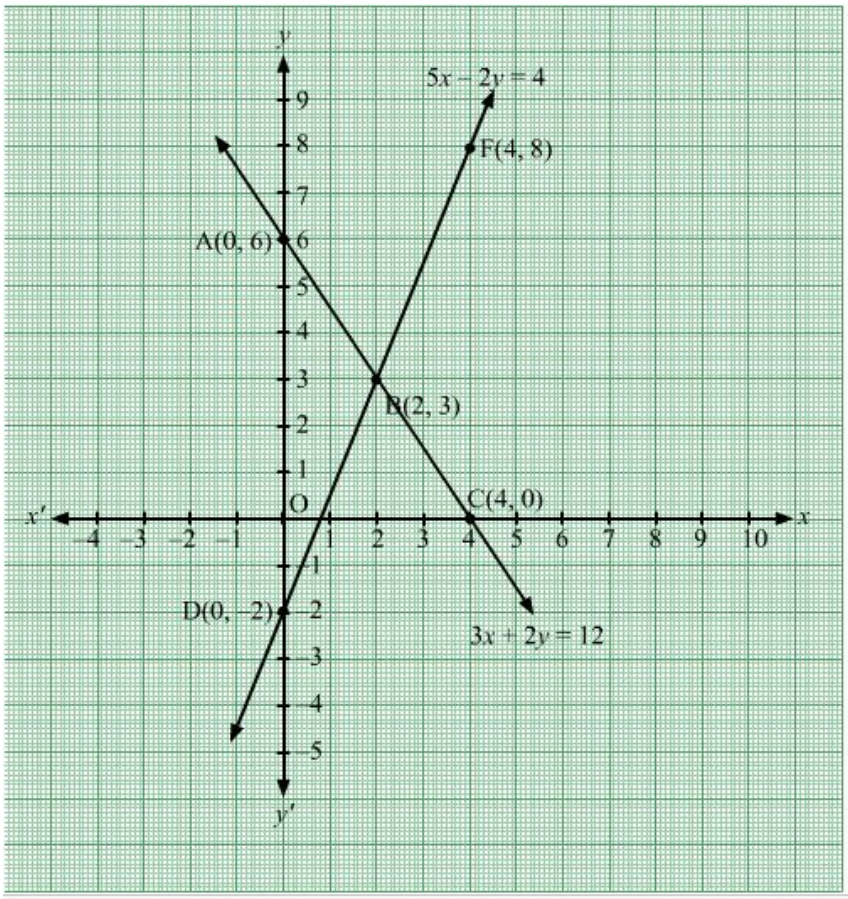
$$\frac{4}{2(m-n)}=\frac{12}{5m-1} \Rightarrow m-6n=-1 \qquad ...(2)$$

Solving (1) and (2) we get, $m = 5, n = 1$

OR

Solve the system of equations graphically: $3x + 2y = 12$, $5x – 2y = 4$

Ans:



From the graph it is clear that, the given lines intersect at (2, 3).
Hence, the solution of the given system of equations is (2, 3).

32. The sum of the first 7 terms of an AP is 182. If its 4th and 17th terms are in the ratio 1 : 5, find the AP.

Ans:

Let a be the first term and d be the common difference of the AP.

$\therefore S_7 = 182$

$\Rightarrow \frac{7}{2}(2a+6d) = 182 \qquad \left\{S_n = \frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow a+3d = 26 \qquad .....(1)$

Also,

$a_4 : a_{17} = 1:5 \qquad$ (Given)

$\Rightarrow \frac{a+3d}{a+16d} = \frac{1}{5} \qquad \left[a_n = a+(n-1)d\right]$

$\Rightarrow 5a+15d = a+16d$

$\Rightarrow d = 4a \qquad .....(2)$

Solving (1) and (2), we get

$a+3\times 4a = 26$

$\Rightarrow 13a = 26$

$\Rightarrow a = 2$

Putting $a = 2$ in (2), we get

$d = 4\times 2 = 8$

Hence, the required AP is 2, 10, 18, 26,……

33. The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square.

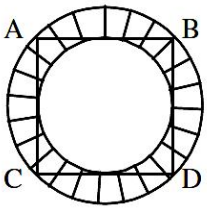
Ans:

Radius of inner circle = 5 cm

Radius of outer circle = $5\sqrt{2}$ cm

Required area = Area of outer circle – Area of inner circle

$\Rightarrow \quad [(5\sqrt{2})^2 - 5^2] = 25\pi \text{ cm}^2$



34. Ram collected the details of marks obtained by students of his class in Mathematics subject. The total number of students is 34. After collecting the data, he analyzed the data and prepared a report on the marks obtained by students of his class. Using this report, he drew the following graph for a particular of marks obtained by students of his class:

Based on the above graph, answer the following questions:

(i) Identify less than type ogive and more than type ogive from the given graph.

(ii) Find the median marks obtained by students using the graph.

(iii) Prepare the frequency distribution table from the graph.

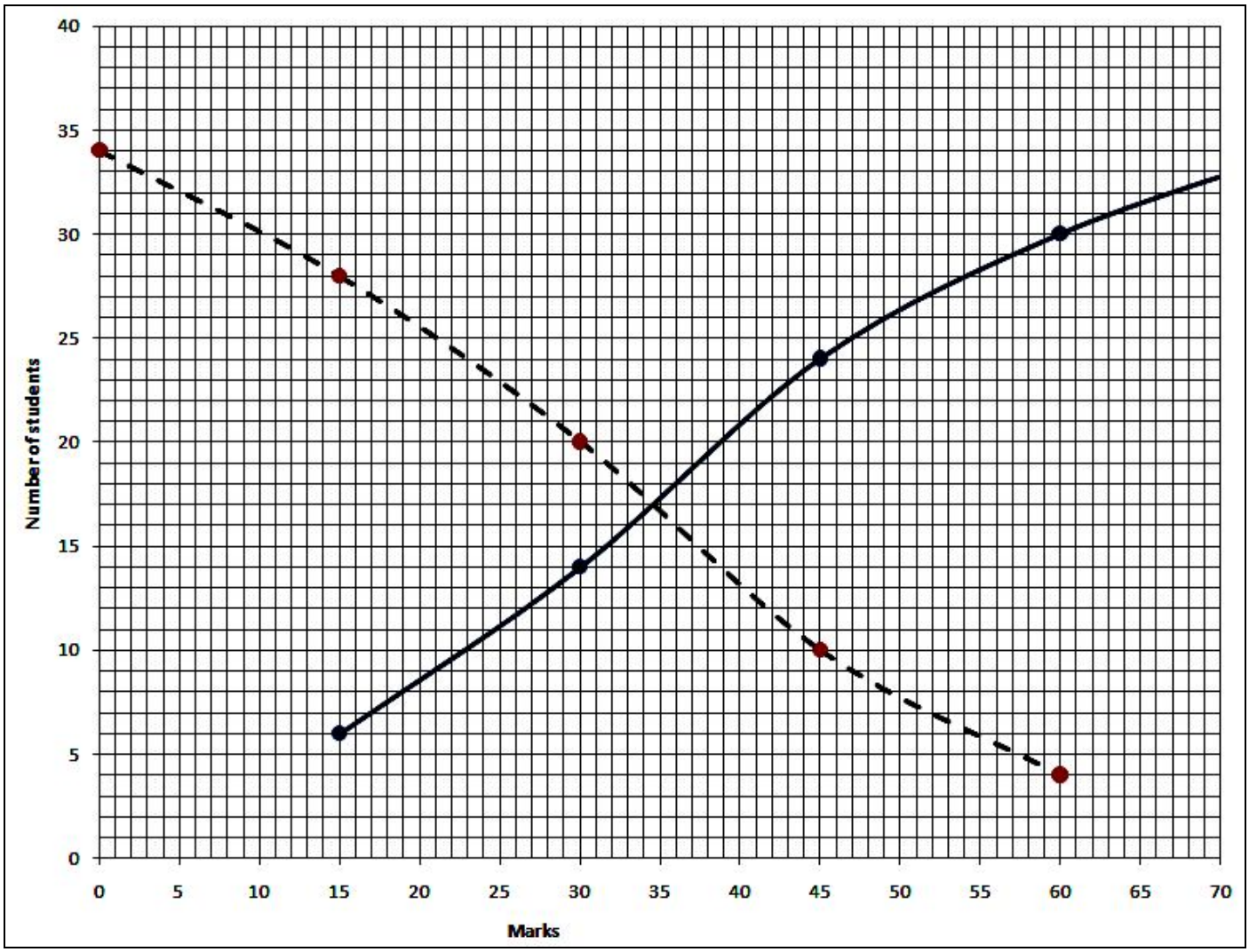
Ans:

(i) Dotted line = more than ogive, dark line = less than ogive

(ii) Median marks obtained by students = 34.5

(iii)

Marks	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75
Number of students	6	8	10	6	4



SECTION – D

Questions 35 to 40 carry 4 marks each.

35. A faster train takes one hour less than a slower train for a journey of 200 km. If the speed of slower train is 10 km/hr less than that of faster train, find the speeds of two trains.

Ans:

Let the speed of faster train be x km/hr

$\therefore$ Speed of slower train = (x – 10) km/hr

$$\frac{200}{x-10}-\frac{200}{x}=1$$

$\Rightarrow$ $x^2 - 10x - 2000 = 0$ $\Rightarrow$ $(x - 50)(x + 40) = 0$

$x = 50, -40$ rejected

$\therefore$ Speed of faster train = 50 km / hr

Speed of slower train = 40 km / hr

OR

Solve for x: $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}, a\neq 0, b\neq 0, x\neq 0$

Ans:

$$\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x} \Rightarrow \frac{1}{a+b+c}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$

$$\frac{-(a+b)}{x(a+b+x)}=\frac{a+b}{ab}$$

$\Rightarrow \quad x^2+(a+b)x+ab=0$

$(x+a)(x+b)=0 \Rightarrow x=-a,-b$

36. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

37. The angle of elevation of the top of a hill at the foot of a tower is 60º and the angle of depression from the top of tower to the foot of hill is 30º. If tower is 50 metre high, find the height of the hill.

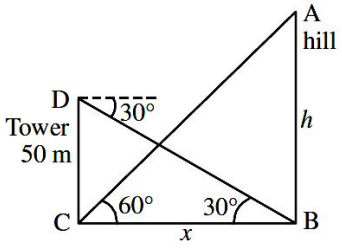
Ans:

In ΔABC, $\frac{h}{x}=\tan 60° \Rightarrow h=x\sqrt{3}$

In ΔBCD, $\frac{50}{x}=\tan 30° \Rightarrow x=50\sqrt{3}$

$\therefore \quad h=150$

$\therefore \quad$ height of hill $= 150$ m



38. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

OR

Draw a triangle ABC with side BC=6.5cm, ∠B=30°, ∠A =105°. Then construct another triangle whose sides are 3/4 times the corresponding sides of the triangle ABC.

39. Water is flowing at the rate of 15km/hour through a pipe of diameter 14cm into a cuboidal pond which is 50m long and 44m wide. In what time will the level of water in the pond rise by 21cm?

Ans:

Given : Diameter of cylindrical pipe = 14 cm

$\Rightarrow$ Radius of pipe = 7 cm = $\frac{7}{100}$ m

Length of cuboidal pond = 50 m

Breadth of cuboidal pond = 44 m

Rate of flow of water = 15 km/hr

Quantity of water flowing through pipe in 1 hour

$$=\frac{22}{7}\times\frac{7}{100}\times\frac{7}{100}\times 15000\,m^3$$

When the height of water level in the pond is 21 cm

Then, Volume of water in the pond $= 50\times 44\times\frac{21}{100}$

$\therefore$ Time in which tank will fill to a height of 21 cm

$$=\frac{\text{Volume of cuboid pond}}{\text{Volume of water flow in 1 hr}}$$

$$=\frac{50\times 44\times\frac{21}{100}}{\frac{22}{7}\times\frac{7}{100}\times\frac{7}{100}\times 15000} \qquad =\frac{50\times 44\times 21\times 7\times 100\times 100}{22\times 7\times 7\times 15000\times 100}$$

$= 2$ hours.

OR

A petrol tank is in the form of a frustum of a cone of height 20 m with diameters of its lower and upper ends as 20 m and 50 m respectively. Find the cost of petrol which can fill the tank completely at the rate of Rs. 70 per litre. Also find the surface area of the tank.

Ans: Capacity of tank = $\pi \times 20 \times (10^2 + 25^2 + 10 \times 25)m^3$

= $\pi \times 20 \times 325\ m^3 = \pi \times 20 \times 325 \times 1000\ l$

Cost of petrol = $\pi \times 20 \times 325 \times 1000 \times 70$ = Rs. 1430000000

Slant height = $\sqrt{20^2 + (25 - 10)^2}$ = 25m

Surface area of tank = $\pi \times 25(10 + 25)m^2 = 2750\ m^2$

40. Find the mean and mode for the following data :

Classes	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	4	8	10	12	10	4	2

Ans: Mean = 42.2, Mode = 45