CLASS X (2019-20)

MATHEMATICS STANDARD(041)

SAMPLE PAPER-10

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is [1]

(a) 240 (b) 1600

(c) 2400 (d) 3600

Ans : (d) 3600

The L.C.M. of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect number.

2. If the sum of the zeroes of the polynomial $\mathrm{f(x)} = 2\mathrm{x}^3 - 3\mathrm{kx}^2 + 4\mathrm{x} - 5$ is 6, then the value of k is [1]

(a) 2 (b) -2

(c) 4 (d) -4

Ans : (c) 4

$$\text{Sum of the zeroes} = \frac{3\mathrm{k}}{2}$$

$$6 = \frac{3\mathrm{k}}{2}$$

$$\mathrm{k} = \frac{12}{3} = 4$$

3. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of $x + y$ is [1]

(a) 10 (b) 11

(c) 12 (d) 13

Ans : (b) 11

The numbers that can be formed are xy and yx. Hence, $(10x + y) + (10y + x) = 11(x + y)$. If this is a perfect square that $x + y = 11$.

4. The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are [1]

(a) 1, 8 (b) $-1, -8$

(c) $-1, 8$ (d) $1, -8$

Ans : (d) $1, -8$

The given equation is

$$x^{2/3} + x^{1/3} - 2 = 0$$

Put $x^{1/3} = y$,

then $y^2 + y - 2 = 0$

$$(y-1)(y+2) = 0$$

$$y = 1$$

or $y = -2$

$$x^{1/3} = 1$$

or $x^{1/3} = -2$

$$x = (1)^3$$

or $x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, -8.

5. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be [1]

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

Ans : (b) 3.5

For an AP, $a_n = a + (n-1)d$

$$= 3.5 + (101 - 1) \times 0$$

[by given conditions]

$$= 3.5$$

6. If the area of the triangle formed by the points $(x, 2x)$, $(-2, 6)$ and (3, 1) is 5 sq units, then x equals [1]

(a) 2/3 (b) 3/5

(c) 3 (d) 5

Ans : (a) 2/3

We have, area = 5 sq units

$$\frac{1}{2}[x(6-1) - 2(1-2x) + 3(2x-6)] = \pm 5$$

$$5x - 2 + 4x + 6x - 18 = \pm 10$$

$$15x = \pm 10 + 20$$

$$15x = 30 \text{ or } 10$$

$$x = \frac{30}{15} \text{ or } \frac{10}{15}$$

$$x = 2 \text{ or } \frac{2}{3}$$

7. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then the angle of elevation of the sun is [1]

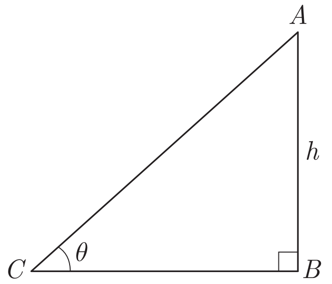
(a) 90° (b) 45°

(c) 30° (d) 75°

Ans : (c) 30°

Let AB be the rod of length h meter.

Let BC be its shadow of length $\sqrt{3}\,h$ meter.



Let angle of elevation of the sun be 'θ'.
In $\Delta\, ABC$,

$$\frac{h}{\sqrt{3}\,h} = \tan\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

8. A sphere is melted and half of the melted liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is [1]

(a) $\left(\frac{4}{3}\right)^{1/3}$ (b) $\left(\frac{8}{3}\right)^{1/3}$

(c) $(3)^{1/3}$ (d) 2

Ans : (b) $\left(\frac{8}{3}\right)^{1/3}$

As per the given conditions,

$$11a^3 = 7 \times \frac{4}{3} \times \pi \times r^3$$

$$\frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

9. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the mean of the last three observation is [1]

(a) $10\frac{1}{3}$ (b) $10\frac{2}{3}$

(c) $11\frac{1}{3}$ (d) $11\frac{2}{3}$

Ans : (c) $11\frac{1}{3}$

We know,

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$$

$$\text{Mean} = \frac{x - x+3 - x+5 - x+7 + x-10}{5}$$

$$9 = \frac{5x+25}{5}$$

$$x = 4$$

So, mean of last three observation is

$$\frac{3x+22}{3} = \frac{12+22}{3} = \frac{34}{3} = 11\frac{1}{3}$$

10. If in a lottery, there are 5 prizes and 20 blanks, then the probability of getting a prize is [1]

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) 1

Ans : (c) $\frac{1}{5}$

$$\text{Required probability} = \frac{5}{25} = \frac{1}{5}$$

(Q.11-Q.15) Fill in the blanks.

11. Two figures having the same shape and size are said to be .... [1]

Ans : congruent

12. Points (3, 2), $(-2, -3)$ and (2, 3) form a .... triangle. [1]

Ans : right angle

or

The distance of the point (x_1, y_1) from the origin is ....

Ans : $\sqrt{x_1^2 + y_1^2}$

13. $\sin^2\theta + \sin^2(90^\circ - \theta) =$ .... [1]

Ans : 1 [**Hint :** $\sin^2(90^\circ - \theta)] = \cos^2\theta$]

14. The tangent to a circle is .... to the radius through the point of contact. [1]

Ans : perpendicular

15. A curve made by moving one point at a fixed distance from another is called .... [1]

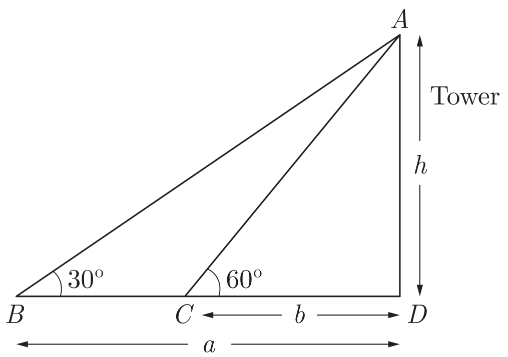
Ans : Circle

(Q.16-Q.20) Answer the following

16. If the angles of elevation of the top of a tower from two points distant a and $b(a > b)$ from its foot and in the same straight line from it are respectively 30° and 60°, then find the height of the tower. [1]

Ans :

Let the height of tower be h. As per given in question we have drawn figure below.



From $\Delta\, ABD$, $\frac{h}{a} = \tan 30^\circ$

$$h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad ...(1)$$

From $\Delta\, ACD$, $\frac{h}{b} = \tan 60^\circ$

$$h = b \times \sqrt{3} = b\sqrt{3} \quad ...(2)$$

From (1) $a = \sqrt{3}\,h$

From (2) $b = \frac{h}{\sqrt{3}}$

Thus $a \times b = \sqrt{3}\,h \times \frac{h}{\sqrt{3}}$

$$ab = h^2$$

$$h = \sqrt{ab}$$

Hence, the height of the tower is $\sqrt{ab}$.

17. The diameter of a wheel is 1.26 m. What the distance covered in 500 revolutions. [1]

Ans :

Distance covered in 1 revolution is equal to circumference of wheel and that is πd.

Distance covered in 500 revolutions

$$= 500 \times \pi \times 1.26$$

$$= 500 \times \frac{22}{7} \times 1.26$$

$$= 1980 \text{ m.} = 1.98 \text{ km}$$

18. The slant height of a bucket is 26 cm. The diameter of upper and lower circular ends are 36 cm and 16 cm. Find the height of the bucket. [1]

Ans :

Given,
Here, $l = 26$ cm, upper radius $= 18$ cm,

lower radius $= 8$ cm

d = difference in radius $= 18 - 8 = 10$ cm.

Let h be the height of bucket

$$h = \sqrt{l^2 - d^2} = \sqrt{(26)^2 - (10)^2}$$
$$= \sqrt{676 - 100} = \sqrt{576} = 24 \text{ cm.}$$

or

A cylinder and a cone have base radii 5 cm and 3 cm respectively and their respective heights are 4 cm and 8 cm. Find the ratio of their volumes.

Ans :

$$\text{Volume of cylinder } = \pi(5)^2 \times 4 \text{ cm}^3 = 100\pi \text{ cm}^3$$

$$\text{Volume of cone } = \frac{1}{3}\pi \times 3^2 \times 8 = 24\pi$$

$$\text{Required ratio } = 100\pi : 24\pi = 25 : 6.$$

19. Consider the following distribution : [1]

Marks Obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
Number of students	63	58	55	51	48	42

(i) Calculate the frequency of the class 30 - 40.
(ii) Calculate the class mark of the class 10 - 25.

Ans :

(i)

Class Interval	**c.f.**	**f**
0-10	63	5
10-20	58	3
20-30	5	4
30-40	51	3
40-50	48	6
50-60	42	42

So, frequency of the class 30 - 40 is 3.

(ii) Class mark of the class : $10 - 25 = \frac{10 + 25}{2}$

$$= \frac{35}{2} = 17.5$$

20. A bag contains cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that number is divisible by both 2 and 3. [1]

Ans :

The numbers divisible by 2 and 3 both

$$= 6, 12, 18, 24$$
$$= 4$$

$$\therefore \quad P \text{ (number divisible by 2 and 3)} = \frac{4}{25}$$

Section B

21. Given that HCF $(306, 1314) = 18$. Find LCM $(306, 1314)$ [2]

Ans :

We have HCF $(306, 314) = 18$

LCM $(306, 1314) = ?$

Let $a = 306$ and $b = 1314$, then we have

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$

Substituting values we have

or, $\text{LCM}(a, b) \times 18 = 306 \times 1314$

or, $LCM\,(a, b) = \frac{306 \times 1314}{18}$

$$LCM(306, 1,314) = 22,338$$

22. If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k. [2]

Ans :

We have $6x^2 - x - k = 0$

Substituting $x = \frac{2}{3}$, we get

$$6\left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$$

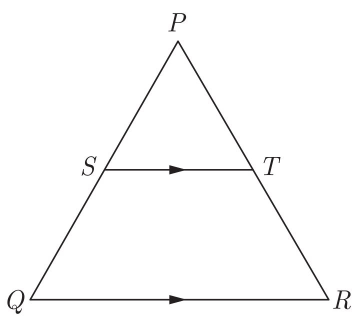
$$6 \times \frac{4}{3} - \frac{2}{3} - k = 0$$

$$k = 6 \times \frac{4}{9} - \frac{2}{3}$$

$$= \frac{24 - 6}{9} = 2$$

Thus $k = 2$.

23. In the given figure, in a triangle $PQR, ST \,||\, QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT. [2]



Ans :

We have $\frac{PS}{SQ} = \frac{3}{5}$

$$\frac{PS}{PS+SQ} = \frac{3}{3+5}$$

$$\frac{PS}{PQ} = \frac{3}{8}$$

According to the question, $ST || QR$, thus

$$\frac{PS}{PQ} = \frac{PT}{PR} \quad \text{(By BPT)}$$

$$PT = \frac{PS}{PQ} \times PR$$

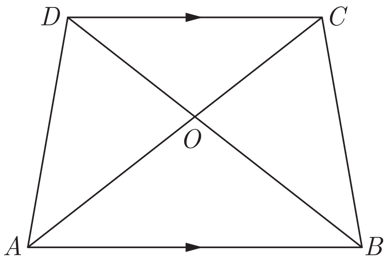
$$= \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

or

$ABCD$ is a trapezium in which $AB || CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans :

As per given condition we have drawn the figure below.



In ΔAOB and ΔCOD, $AB || CD$,

Thus $\angle OAB = \angle DCO$

and $\angle OBA = \angle ODC$ (Alternate angles)

By AA similarity we have

$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}. \quad \text{Hence Proved}$$

24. There are 60 students in a class among which 30 are boys. In another class there are 50 students among which 25 of them are boys. If one from each class is selected, [2]

(a) What is the probability of both being girls ?

(b) What is the probability of having atleast one girl?

Ans :

Total number of students in the first class = 60

No. of boys = 30

No. of girls = 30

Total number of students in the second class = 50

No. of boys = 25

No. of girls = 25

(a) Probability of both being girls

$$= \frac{30 \times 25}{60 \times 50} = \frac{750}{3000} = \frac{1}{4}$$

(b) Probability of at least one girl

$$= \frac{30 \times 25 + 30 \times 25 + 30 \times 25}{3000}$$

$$= \frac{2250}{3000} = \frac{3}{4}$$

25. Find the mean of the following distribution : [2]

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	5	4	1	6	4

Ans :

x_i	f_i	$x_i f_i$
3	5	15
9	4	36
15	1	15
21	6	126
27	4	108
Total	$\sum f_i = 20$	$\sum x_i f_i = 300$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{300}{20} = 15$$

or

Find the mode of the following distribution :

Classes	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Ans :

Here, Modal class = 35 − 40

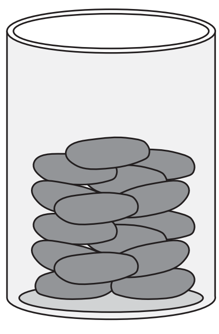
$$l = 35,\ f_1 = 50,\ f_2 = 42,\ f_0 = 34,\ h = 5$$

$$\text{Mode} = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

$$= 35 + \frac{50 - 34}{100 - 34 - 42} \times 5$$

$$= 35 + \frac{16 \times 5}{24} = 38.33$$

26. A gulab jamun, contains sugar syrup upto about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm. [2]



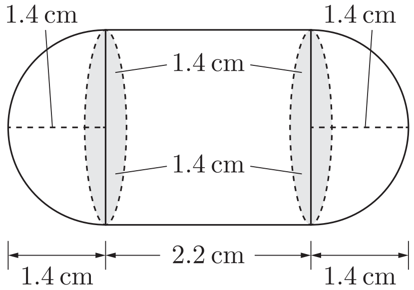
Ans :

Radius of cylindrical portion and hemispherical portion of a gulab jamun

$$= \frac{2.8}{2} = 1.4 \text{ cm}$$

Length of cylindrical portion

$$= 5 - 1.4 - 1.4 = 2.2 \text{ cm}$$



Now, Volume of one gulab jamun = Volume of cylindrical part $+2 \times$ Volume of hemispherical part

$$= \pi(1.4)^2 \times 2.2 + 2 \times \frac{2}{3}\pi(1.4)^3$$

$$= \frac{22}{7} \times (1.4)^2\left[2.2 + \frac{4}{3} \times 1.4\right]$$

$$= \frac{22}{7} \times 1.96 \times \frac{12.2}{3}$$

$$= \frac{75.152}{3} \text{ cm}^2$$

Volume of 45 gulab jamun

$$= 45 \times \frac{75.152}{3} = 1127.28 \text{ cm}^2$$

Volume of syrup in 45 gulab jamuns

$$= 30\% \text{ of } 1127.28$$

$$= \frac{30}{100} \times 1127.28 = 338.18 \text{ cm}^3$$

$$= 338 \text{ cm}^3 \text{ (approx)}.$$

Section C

27. Find the HCF and LCM of 510 and 92 and verify that HCF $\times$ LCM = Product of two given numbers. [3]

Ans :

Finding prime factor of given number we have,

$$92 = 2^2 \times 23$$

$$510 = 30 \times 17 = 2 \times 3 \times 5 \times 17$$

$$\text{HCF } (510, 92) = 2$$

$$\text{LCM } (510, 92 = 2^2 \times 23 \times 3 \times 5 \times 14$$

$$= 23460$$

$$\text{HCF } (510, 92) \times \text{LCM } (510, 92)$$

$$= 2 \times 23460 = 46920$$

Product of two numbers $= 510 \times 92 = 46920$

Hence, HCF$\times$LCM = Product of two numbers

or

Show that any positive odd integer is of the form $6q+1, 6q+3$ or $6q+5$, where q is some integer.

Ans :

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r$$

Take $b = 6$, then $0 \le r < 6$ because $0 \le r < b$,

Thus $a = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

Here $6q$, , $6q+2$ and $6q+4$ are divisible by 2 and so $6q$, $6q+2$ and $6q+4$ are even positive integers.
But $6q+1, 6q+3, 6q+5$ are odd, as they are not divisible by 2.
Thus any positive odd integer is of the form $6q+1, 6q+3$ or $6q+5$.

28. Solve for x : $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ [3]

Ans :

We have $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$$\sqrt{3}x^2 - [3\sqrt{2} - \sqrt{2}]x - 2\sqrt{3} = 0$$

$$\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$$

$$\sqrt{3}x^2 - \sqrt{3}\sqrt{3}\sqrt{2}x + \sqrt{2}x - \sqrt{2}\sqrt{2}\sqrt{3} = 0$$

$$\sqrt{3}x[x - \sqrt{3}.\sqrt{2}] + \sqrt{2}[x - \sqrt{2}\sqrt{3}] = 0$$

$$\sqrt{3}x[x - \sqrt{6}] + \sqrt{2}[x - \sqrt{6}] = 0$$

$$(x - \sqrt{6})(\sqrt{3}x + \sqrt{2}) = 0$$

Thus $x = \sqrt{6} = -\sqrt{\frac{2}{3}}$

29. The sum of n terms of an A.P. is $3n^2 + 5n$. Find the A.P. Hence find its 15^{th} term. [3]

Ans :

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $S_n = 3n^2 + 5m$

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$

$$= 3(n^2 + 1 - 2n) + 5n - 5$$

$$= 3n^2 + 3 - 6n + 5n - 5$$

$$= 3n^2 - n - 2$$

$$a_n = S_n - S_{n-1}$$

$$= 3n^2 + 5n - (3n^2 - n - 2)$$

$$= 6n + 2$$

Thus A.P. is 8, 14, 20, ...

Now $a_{15} = a + 14d = 8 + 14(6) = 92$

or

Find the 20^{th} term of an A.P. whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a + 2d = 7$ (1)

$$a_7 = 3a_3 + 2$$

$$a + 6d = 3 \times 7 + 2 = 23 \quad (2)$$

Solving (1) and (2), we have

$$4d = 16 \Rightarrow d = 4$$

$$a + 8 = 7 \Rightarrow a = -1$$

$$a_{20} = a + 19d = -1 + 19 \times 4 = 75$$

$$a_1 = a + (n-1)d$$

$$= -1 + 4n - 4$$

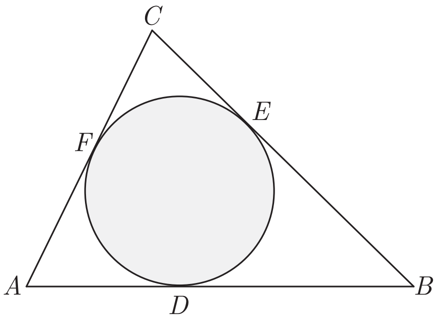
$$= 4n - 5.$$

Hence n^{th} term is $4n - 5$

30. A circle is inscribed in a ΔABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF. [3]

Ans :

As per question we draw figure shown below.



We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$12 = 10 - x + 8 - x$

$2x = 18 - 12 = 6$

or $x = 3$

Now $AD = 3$ cm,

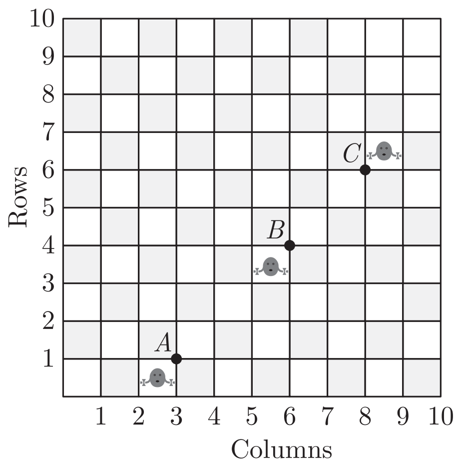
$BE = 10 - 3 = 7$ cm

and $CF = 8 - 3 = 5$

31. Given figure shows the arrangement of desks in a classroom. Ashima, Bharti and Camella are seated at $A(3,1)$, $B(6,4)$ and $C(8,6)$ respectively. [3]

(i) Do you think they are seated in a line? Give reasons for your answer.

(ii) Which mathematical concept is used in the above problem?



Ans :

(i) Using distance formula, we have

$$AB = \sqrt{(6-3)^2 + (4-1)^2}$$
$$= \sqrt{(3)^2 + (3)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units.}$$
$$BC = \sqrt{(8-6)^2 + (6-4)^2}$$
$$= \sqrt{(2)^2 + (2)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$
$$AC = \sqrt{(8-3)^2 + (6-1)^2}$$
$$= \sqrt{(5)^2 + (5)^2} = \sqrt{25+25}$$
$$= \sqrt{50} = 5\sqrt{2} \text{ units}$$

Since, $AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC$

$\therefore$ A, B and C are collinear.

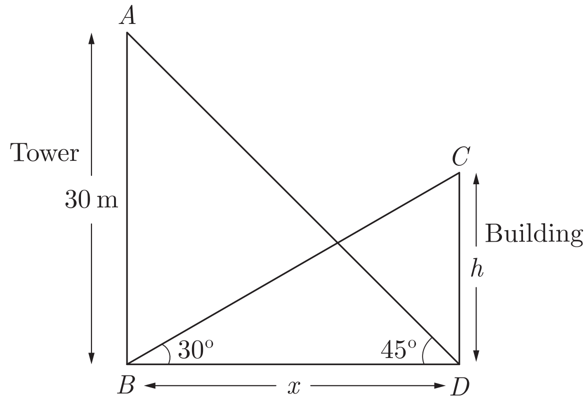
Thus, Ashima, Bharti and Camella are seated in a line.

(ii) Co-ordinate Geometry.

32. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building. [3]

Ans :

Let the height of the building be $AB = h$ m. and distant between tower and building be, $BD = x$ m. As per given in question we have drawn figure below.



In ΔABD $\tan 45^\circ = \frac{AB}{BD}$

$$1 = \frac{30}{x}$$

$$x = 30 \quad ...(1)$$

Now in ΔBDC,

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\sqrt{3}\,h = x \Rightarrow h = \frac{x}{\sqrt{3}} \quad ...(2)$$

From (1) and (2), we get

$$h = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m.}$$

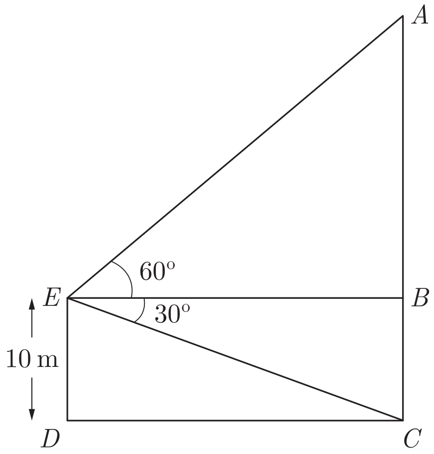
Therefore height of the building is $10\sqrt{3}$ m

or

A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60^o and the angle of depression of the base of hill as 30^o. Find the distance of the hill from the ship and the height of the hill.

Ans :

As per given in question we have drawn figure below. Here AC is height of hill and man is at E. $ED = 10$ is height of ship from water level. As per given in question we have drawn figure below.



In ΔBCE, $BC = 10$ m and

$$\angle BEC = 30^o$$

Now $$\tan 30^o = \frac{BC}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{BE}$$

$$BE = 10\sqrt{3}$$

Since $BE = CD$, distance of hill from ship

$$CD = 10\sqrt{3}\text{ m} = 10 \times 1.732 \text{ m}$$
$$= 17.32 \text{ m}$$

Now in ΔABE, $\angle AEB = 60^o$

where $AB = h$, $BE = 10\sqrt{3}$ m

and $$\angle AEB = 60^o$$

Thus $$\tan 60^o = \frac{AB}{BE}$$

$$\sqrt{3} = \frac{AB}{10\sqrt{3}}$$

$$AB = 10\sqrt{3} \times \sqrt{3} = 30\,m$$

Thus height of hill $AB + 10 = 40$ m

33. A hemispherical bowl of internal diameter 36 cm contains liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find. the height of the each bottle, if 10% liquid is wasted in this transfer. [3]

Ans :

$$\text{Volume of bowl} = \frac{2}{3}\pi r^3$$

$$\text{Volume of liquid in bowl} = \frac{2}{3}\pi \times (18)^3 \text{ cm}^3$$

$$\text{Volume of one after wastage} = \frac{2}{3}\pi(18)^3 \times \frac{90}{100} \text{ cm}^3$$

$$\text{Volume of one bottle} = \pi r^2 h$$

Volume of liquid in 72 bottles

$$= \pi \times (3)^2 \times h \times 72 \text{ cm}^2$$

Volume of bottles = volume in liquid after wastage

$$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}$$

$$h = \frac{\frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

Hence, the height of bottle = 5.4cm

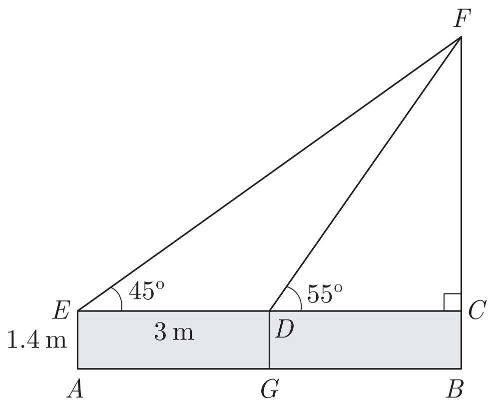
34. A boy, 1.4 metre tall standing at the edge of a river bank sees the top of a tree on the edge of the other bank at an elevation of 55°. Standing back by 3 metre, he sees it at elevation of 45°. [3]

(a) Draw a rough figure showing these facts.

(b) How wide is the river and how tall is the tree ? $[\sin 55^\circ = 0.8192, \cos 55^\circ = 0.5736, \tan 55^\circ = 1.4281]$

Ans :

(a) The rough sketch is as follows :



(b) Here, BF represents the tree, and CD represents the river.

DG is the initial position of the boy and AE is the new position.

Here, $AE = DG = BC = 1.4$ m

If $DC = x$ m

then $EC = (x + 3)$ m

In right ΔECF,

$$\tan 45^\circ = \frac{CF}{EC}$$

$$1 = \frac{CF}{x + 3}$$

$$CF = (x + 3)$$

In right ΔDCF,

$$\tan 55^\circ = \frac{CF}{DC}$$

$$1.4281 = \frac{x + 3}{x}$$

$$1.4281x = x + 3$$

$$0.4281x = 3$$

$$x = \frac{3}{0.4281} = 7$$

$$\text{Width of the river} = CD = 7 \text{ m}$$

$$\text{Height of the tree} = BF + BC + CF$$
$$= (1.4 + 7 + 3) = 11.4 \text{ m}$$

Section D

35. Obtain all other zeroes of the polynomial $x^4 + 6x^3 + x^2 - 24x - 20$, if two of its zeroes are $+2$ and -5. [4]

Ans :

$$\begin{array}{r} x^2+3x+2 \\ x^2+3x-10 \overline{)x^4+6x^3+x^2-24x-20} \\ \underline{x^4+3x^3-10x^2} \\ 3x^3+11x^2-24x-20 \\ \underline{3x^3+9x^2-30x} \\ 2x^2+6x-20 \\ \underline{2x^2+6x-20} \\ 0 \end{array}$$

As $x=2$ and -5 are the zeroes of $x^4+6x^3+x^2-24x-20$.

So $(x-2)$ and $(x+5)$ are two factors of $x^4+6x^3+x^2-24x-20$ and the product of factors is

$$(x-2)(x+5) = x^2+3x-10=0$$

Dividing $x^4+6x^3+x^2-24x-20$ by $x^2+3x-10$

$$= x^4+6x^3+x^2-24x-20$$
$$= (x^2+3x-10)(x^2+3x+2)$$
$$= (x-2)(x+5)(x+2)(x+1)$$

Hence other two zeroes are -2 and 1.

or

Obtain all other zeroes of the polynomial $4x^4+x^3-72x^2-18x$, if two of its zeroes are $3\sqrt{2}$ and $-3\sqrt{2}$.

Ans :

As $3\sqrt{2}$ and $-3\sqrt{2}$ are the zeroes of $4x^4+x^3-72x^2-18x$, So $(x-3\sqrt{2})$ and $(x+3\sqrt{2})$ are its two factors

Now,$(x-3\sqrt{2})(x+3\sqrt{2}) = 0$

or, $x^2-18 = 0$

On Factorising quotient $4x^2+2$

We get, $x = 0$ and $\frac{1}{4}$

$$= (x^2-18)x(4x+1)$$
$$= (x-3\sqrt{2})(x+3\sqrt{2})(x)(4x+1)$$

Hence, other two zeroes are 0 and $\frac{-1}{4}$.

36. A and B are two points 150 km apart on a highway. Two cars start A and B at the same time. If they move in the same direction they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hours. Find their speeds. [4]

Ans :

Let the speed of the car I from A be x km/hr. Speed of the car II from B be y km/hr.

Same Direction :

Distance covered by car I= 150 +(distance covered by car II)

$$15x = 150+15y$$
$$15x-15y = 150$$
$$x-y = 10 \qquad ...(1)$$

Opposite Direction :

Distance covered by car I + distance covered by car II

$$= 150 \text{ km}$$
$$x+y = 150 \qquad ...(2)$$

from equation (1) and (2), we have

$$x = 80 \text{ km}$$
$$y = 70$$

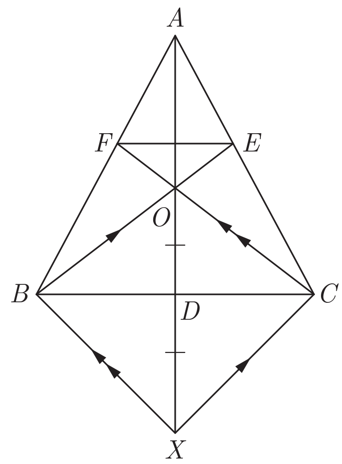
i.e. Speed of the car I from $A = 80$ km/hr. and speed of the car II from $B = 70$ km/hr.

37. In $\Delta ABC, AD$ is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure. [4]

Prove that :

(1) $EF || BC$

(2) $AO : AX = AF : AB$



Ans :

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE || XC$ and $BX || CF$.

In ΔABX, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \qquad ..(1)$$

In ΔAXC, $$\frac{AE}{EC} = \frac{AO}{OX} \qquad ...(2)$$

From equation (1) and (2), we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

By converse of BPT we have

$$EF \, || \, BC$$

From (1) we get $\frac{OX}{OA} = \frac{FB}{AF}$

$$\frac{OX+OA}{OA} = \frac{FB+AF}{AF}$$
$$\frac{AX}{OA} = \frac{AB}{AF}$$
$$\frac{AO}{AX} = \frac{AF}{AB}$$

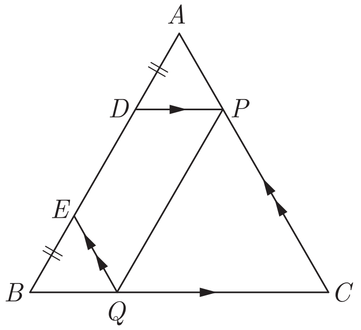
Thus $AO : AX = AF : AB$ Hence Proved

or

Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP || BC$ and $EQ || AC$, then prove that $PQ || AB$.

Ans :

As per given condition we have drawn the figure below.



In ΔABC, $DP \parallel BC$

By BPT we have $\dfrac{AD}{DB} = \dfrac{AP}{PC}$, ...(1)

Similarly, in ΔABC, $EQ \parallel AC$

$$\frac{BQ}{QC} = \frac{BE}{EA} \quad ...(2)$$

From figure, $EA = AD + DE$

$= BE + ED$ $\quad (\;BE = AD)$

$= BD$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \quad ...(3)$$

From (1) and (3), we get

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

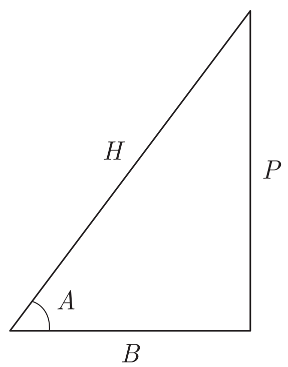
By converse of BPT,

$PQ \parallel AB$ Hence Proved

38. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$. [4]

Ans :

Consider the triangle shown below.



Let $\tan A = \dfrac{P}{B}$ and $\sec A = \dfrac{H}{B}$

$$H^2 = P^2 + B^2$$

Now $1 + \tan^2 A = 1 + \left(\dfrac{P}{B}\right)^2 = 1 + \dfrac{P^2}{B^2}$

$$= \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2} = \left(\frac{H}{B}\right)^2$$

$= \sec^2 A$ Hence Proved.

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement.

or

Given that $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, find the values of $\tan 75^\circ$ and $\tan 90^\circ$ by taking suitable values of A and B.

Ans :

We have $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(i) $\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 \cdot \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{3 + 2\sqrt{3} + 1}{(\sqrt{3})^2 - (1)^2} = \frac{4 + 2\sqrt{3}}{2}$$

Hence $\tan 75^\circ = 2 + \sqrt{3}$

(ii) $\tan 90^\circ = \tan(60^\circ + 30^\circ)$

$$= \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ}$$

$$= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$$

Hence, $\tan 90^\circ = \infty$

39. Find the values of k for which the points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear. [4]

Ans :

If three points are collinear, then area covered by given points must be zero.

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$[(k+1)(2k+3-5k) + 3k(5k-2k) +$$

$$+(5k-1)(2k-2k-3) = 0$$

$$-3k^2 + 3k - 3k + 3 + 9k^2 - 15k + 3 = 0 = 0$$

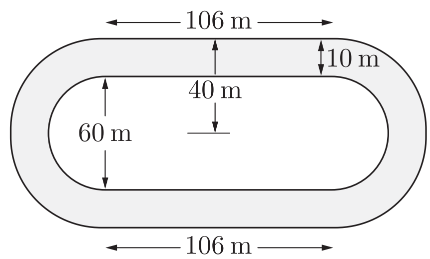
$$6k^2 - 15k + 6 = 0$$

$$2k^2 - 5k + 2 = 0$$

$$(k-2)(2k-1) = 0$$

Thus $k = 2$ or $k = \dfrac{1}{2}$

40. Figure depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide everywhere, find the area of the track. [4]



Ans :

Width of the inner parallel lines $= 60$ m
And the width of the outer lines $= 40 \times 2 = 80$ m

Radius of the inner semicircles $= \frac{60}{2} = 30$ m

Radius of the other semicircles $= \frac{80}{2} = 40$ m

Area of inner rectangle $= 106 \times 60 = 3180$ m^2

Area of outer rectangle $= 106 \times 80 = 4240$ m^2.

Area of the inner semicircle

$$= 2\frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{19800}{7} \text{ m}^2$$

Area of outer semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{35200}{7} \text{ m}^2$$

Area of racing track

= (area of outer rectangle + area of outer semicircles)

– (area of inner rectangle + area of inner semicircles)

$$= 4240 + \frac{35200}{7} - \left(\frac{3180 + 19800}{7}\right)$$

$$= 1060 + \frac{15400}{7} = \frac{7420 + 15400}{7}$$

$$= \frac{22820}{7} = 3260 \text{ m}^2$$

Hence, area of track is 3260 m^2