CLASS X (2019-20)
MATHEMATICS STANDARD(041)
SAMPLE PAPER-8

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

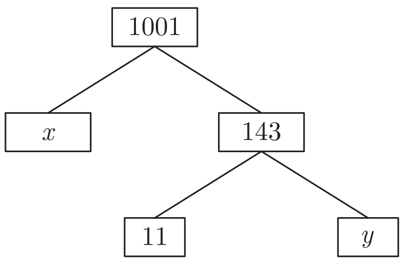
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The values of x and y is the given figure are [1]

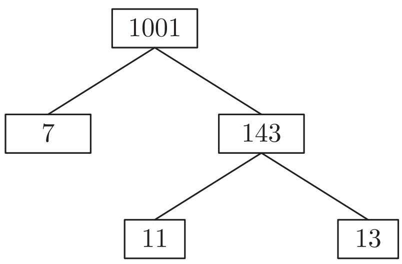


(a) 7, 13 (b) 13, 7

(c) 9, 12 (d) 12, 9

Ans : (a) 7, 13

Given number is 10001. Then, the factor tree of 1001 is given as below



$$1001 = 7 \times 11 \times 13$$

By comparing with given factor tree, we get

$$x = 7,\ y = 13$$

2. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is [1]

(a) 2 (b) -2

(c) 4 (d) -4

Ans : (c) 4

$$\text{Sum of the zeroes} = \frac{3k}{2}$$

$$6 = \frac{3k}{2}$$

$$k = \frac{12}{3} = 4$$

3. If $3x + 4y : x + 2y = 9 : 4$, then $3x + 5y : 3x - y$ is equal to [1]

(a) 4 : 1 (b) 1 : 4

(c) 7 : 1 (d) 1 : 7

Ans : (c) 7 : 1

$$\frac{3x+4y}{x+2y} = \frac{9}{4}$$

Hence, $12x + 16y = 9x + 18y$

or $3x = 2y$

$$x = \frac{2}{3}y$$

Substitute $x = \frac{2}{3}y$ in the required expression.

$$\frac{3\frac{2}{3}y + 5y}{3\frac{2}{3}y - y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

4. The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has [1]

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Ans : (c) no real roots

Given equation is, $2x^2 - \sqrt{5x} + 1 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get $a = 2,\ b = -\sqrt{5}$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$$= (-\sqrt{5})^2 - 4 \times (2) \times (1)$$

$$= 5 - 8 = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has no real roots i.e., imaginary roots.

5. There are 60 terms is an A.P. of which the first term is 8 and the last term is 185. The 31st term is [1]

(a) 56 (b) 94

(c) 85 (d) 98

Ans : (d) 98

Let d be the common difference;

Then 60th term, $= 8 + 59d = 185$

$$59d = 177$$
$$d = 3$$
$$a_n = a + (n-1)d$$

Hence, 31^{th} term $= 8 + 30 \times 3 = 98$

6. The point on the X-axis which if equidistant from the points $A(-2, 3)$ and $B(5, 4)$ is [1]

(a) (0, 2) (b) (2, 0)

(c) (3, 0) (d) $(-2, 0)$

Ans : (b) (2, 0)

Let $P(x, 0)$ be a point on X-axis such that,

$$AP = BP$$
$$AP^2 = BP^2$$
$$(x+2)^2 + (0-3)^2 = (x-5)^2 + (0+4)^2$$
$$x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$$
$$14x = 28$$
$$x = 2$$

Hence, required point $= (2, 0)$

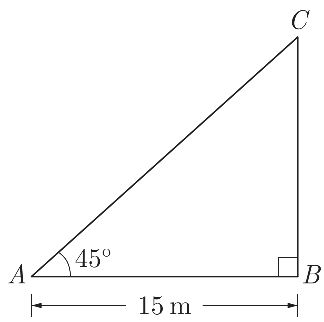
7. The height of a tree, if it casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45°, is [1]

(a) 10 m (b) 14 m

(c) 8 m (d) 15 m

Ans : (d) 15 m

Let BC be the tree of height h meter.
Let AB be the shadow of tree.



In ΔABC $,CB = 90°$

$$\frac{BC}{BA} = \tan 45°$$
$$BC = AB = 15 \text{ m}$$

8. Volume of a spherical shell is given by [1]

(a) $4\pi(R^2 - r^2)$ (b) $\pi(R^3 - r^3)$

(c) $4\pi(R^3 - r^3)$ (d) $\frac{4}{3}\pi(R^3 - r^3)$

Ans : (d) $\frac{4}{3}\pi(R^3 - r^3)$

$$\text{Volume of spherical shell} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(R^3 - r^3)$$

9. The mean of discrete observations y_1, y_2 .... y_n is given by [1]

Ans :

(a) $\frac{\sum_{i=1}^{n} y_i}{n}$ (b) $\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$

(c) $\frac{\sum_{i=1}^{n} y_i f_i}{n}$ (d) $\frac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

Ans : (a) $\frac{\sum_{i=1}^{n} y_i}{n}$

10. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is [1]

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$

(c) $\frac{4}{11}$ (d) 0

Ans : (c) $\frac{4}{11}$

Required probability $= \frac{1+2+1}{11} = \frac{4}{11}$

(Q.11-Q.15) Fill in the blanks.

11. Two polygons of the same number of sides are similar, if all the corresponding angles are .... [1]

Ans : equal

12. Points (1, 5), (2, 3) and $(-2, -11)$ are .... [1]

Ans : Non-collinear

or

The value of the expression $\sqrt{x^2 + y^2}$ is the distance of the point $P(x, y)$ from the ....

Ans : Origin

13. The value of $\sin A$ or $\cos A$ never exceeds .... [1]

Ans : 1

14. Tangent is perpendicular to the .... through the point of contact. [1]

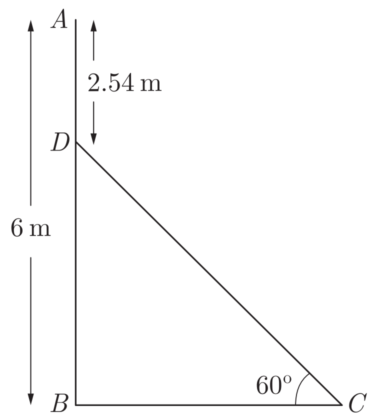
Ans : radius

15. Two circles are drawn with same centre then the .... circle have bigger radius. [1]

Ans : Outer

(Q.16-Q.20) Answer the following

16. In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If $AD = 2.54$ m, find the length of ladder. (use $\sqrt{3} = 1.73$) [1]



Ans :

We have $AD = 2.54$ m

$$DB = 6 - 2.54 = 3.46 \text{ m}$$

In ΔBCD, $\angle B = 90^\circ$

$$\sin 60^\circ = \frac{BD}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

$$DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46}{1.73} = 4$$

Thus length of ladder is 4 m.

17. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring. [1]

Ans :

Circumference of the outer circle $2\pi r_1 = 88$ cm

$$r_1 = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

Circumference of the outer circle $2\pi r_2 = 66$ cm

$$r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm} = 10.5 \text{ cm}$$

Width of the ring

$$r_1 - r_2 = 14 - 10.5 \text{ cm} = 3.5 \text{ cm}$$

18. Volume of two spheres are in the ratio 64 : 27, find the ratio of their surface areas. [1]

Ans :

$$\frac{\text{Volume of I}^{\text{st}}\text{sphere}}{\text{Volume of II}^{\text{nd}}} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

Ratio of their surface areas

$$\frac{4A_1/A_2\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9}$$

or

Find the volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

Ans :

Given,

Edge of the cube $= 4.2$ cm.

Height of the cone, $h = 4.2$ cm.

Radius of the cone, $r = \frac{4.2}{2} = 2.1$ cm.

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2$$

$$= 19.4 \text{ cm}^3$$

19. Following distribution gives cumulative frequencies of 'more than type' : [1]

Marks obtained	**Marks obtained 5**	**More than of equal to 10**	**More than or equal to 15**	**More than of equal to 20**
Number of student (cummulative frequency)	30	23	8	2

Change the above data to a continuous grouped frequency distribution.

Ans :

C.I	5-10	10-15	15-20	20-25
f	7	15	6	2

20. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of getting neither a red card nor a queen. [1]

Ans :

Given, Total number of cards $= 52$

Number of red cards $= 26$

Number of queens which are not red $= 2$

$\therefore$ Cards which are neither red nor queen

$$= 52 - [26 + 2]$$

$$= 24$$

$\therefore$ Required Probability $= \frac{24}{52}$

$$= \frac{6}{13}$$

Section B

21. Find the HCF and LCM of 90 and 144 by the method of prime factorization. [2]

Ans :

We have $90 = 9 \times 10$

$$= 2 \times 3^2 \times 5$$

and $144 = 16 \times 9$

$$= 2^4 \times 3^2$$

$$\text{HCF} = 2 \times 3^2 = 18$$

$$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$$

22. Find the roots of the quadratic equation $\sqrt{3}x^2 - 2x - \sqrt{3}$. [2]

Ans :

We have $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$

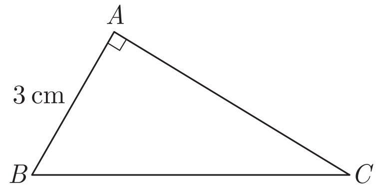
$$\sqrt{3}x^2 - 3x + x - \sqrt{3} = 0$$

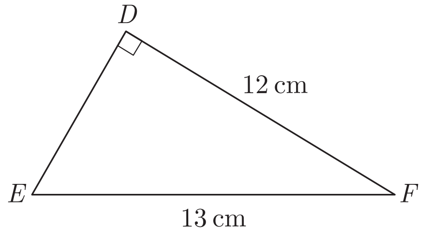
$$\sqrt{3}x(x - \sqrt{3}) + 1(x - \sqrt{3}) = 0$$

$$(x - \sqrt{3})(\sqrt{3} + 1) = 0$$

Thus $x = \sqrt{3}, \frac{-1}{\sqrt{3}}$

23. Given $\Delta ABC \sim \Delta DEF$, find $\frac{\Delta ABC}{\Delta DEF}$ [2]





Ans :

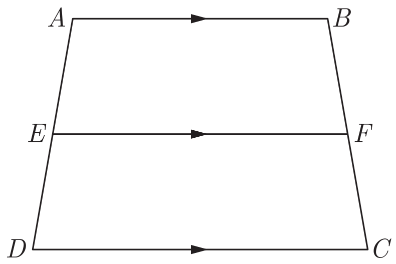
In ΔDEF, we have

$$DE = \sqrt{(13)^2 - (12)^2}$$
$$= \sqrt{169 - 144} = \sqrt{25} = 5$$

Thus $\dfrac{ar\Delta ABC}{ar\Delta DEF} = \left(\dfrac{AB}{DE}\right)^2 = \left(\dfrac{3}{5}\right)^2 = \dfrac{9}{25}$

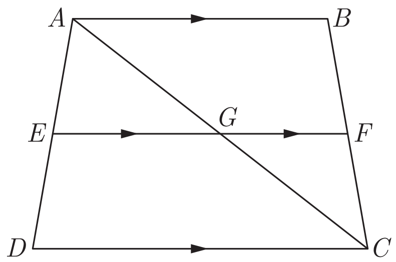
or

In the given figure, if $ABCD$ is a trapezium in which $AB || CD || EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans :

We draw, AC intersecting EF at G as shown below.



In $\Delta CAB, GF || AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \qquad ...(1)$$

In ΔADC, $EG || DC$, thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \qquad ...(2)$$

From equations (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC}.$$ Hence Proved.

24. There are two small boxes A and B. In A, there are 9 white beads and 8 black beads. In B, there are 7 white and 8 black beads. We want to take a bead from a box. [2]

(a) What is the probability of getting a white bead from a box?

(b) A white bead and a black bead are added to box B and then a bead is taken from it. What is the probability of getting a white bead from it ?

Ans :

Total number of beads in box $A = 9W + 8B = 17$
Total number of beads in box $B = 7W + 8B = 15$

(a) P (white bead from box A) $= \dfrac{9}{17}$

P (white bead from box B) $= \dfrac{7}{15}$

$\therefore$ P (a white bead from each box)

$$= \frac{9}{17} \times \frac{7}{15} = \frac{21}{85}$$

(b) When a white bead and a black bead are added to box B, then
No. of white beads in box $B = 7W + 1W = 8W$
No. of black beads in box $B = 8B + 1B = 9B$

$\therefore$ Total number of beads in box $B = 8W + 9B = 17$

Hence, P (white bead from box B) $= \dfrac{8}{17}$

25. Find the value of λ, if the mode of the following data is 20 :
15, 20, 25, 18, 13, 15, 25, 15, 18, 17, 20, 25, 20, λ, 18. [2]

Ans :

Writing the data as discrete frequency distribution, we get

x_i	f_i
13	1
15	3
17	1
18	3
20	3
λ	1
25	3

For 20 to be mode of the frequency distribution, $\lambda = 20$.

or

Find the unknown values in the following table :

Class Interval	Frequency	Cumulative Frequency
0-10	5	5
10-20	7	x_1
20-30	x_2	18
30-40	5	x_3
40-50	x_4	30

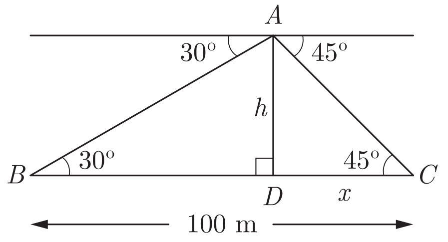
Ans :

$$x_1 = 5 + 7 = 12$$
$$x_2 = 18 - 12 = 6$$
$$x_3 = 18 + 5 = 23$$
and $$x_4 = 30 - 23 = 7$$

26. Two ships are approaching a light-house from opposite directions. The angle of depression of two ships from top of the light-house are 30° and 45°. If the distance between two ships is 100 m, find the height of light-house. [2]

Ans :

Let AD be the height (h meter) of the light-house and BC is the distance between the ships.



Given, $BC = 100\text{ m}$

In right ΔADC,

$$\tan 45^\circ = \frac{AD}{DC}$$

$$1 = \frac{h}{DC}$$

$$DC = h \qquad ...(1)$$

In right ΔADB,

$$\tan 30^\circ = \frac{AD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{100-DC} = \frac{h}{100-h}$$

$$100 - h = h\sqrt{3}$$

$$100 = h + h\sqrt{3} = h(1+\sqrt{3})$$

$$h = \frac{100}{1+\sqrt{3}} = \frac{100}{2.732} = 36.60$$

$\therefore$ Height of tower $= 36.60\text{ m}$

Section C

27. Use Euclid division lemma to show that the square of any positive integer cannot be of the form $5m+2$ or $5m+3$ for some integer m. [3]

Ans :

Let a be any positive integer, then by Euclid's division algorithm a can be written as

$$a = bq + r,\ 0 \le r < b \text{ and } q \in \omega$$

Take $b = 5$, then $0 \le r < 5$ because $0 \le r < b$

Thus $a = 5q,\ 5q+1,\ 5q+2,\ 5q+3$ and $5q+4$,

Now $a^2 = (5q)^2 = 25q^2 = 5(5q^2) = 5m$

$$a^2 = (5q+1)^2 = 25q^2 + 10q + 1 = 5m+1$$

$$a^2 = (5q+2)^2 = 25q^2 + 20q + 4 = 5m+4$$

Similarly $a^2 = (5q+3)^2 = 5m+4$

and $a^2 = (5q+4)^2 = 5m+1$

Thus square of any positive integer cannot be of the form $5m+2$ or $5m+3$.

or

Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Ans :

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3\times 3 = 3^2$$

$$12 = 2\times 2\times 3 = 2^2\times 3$$

$$15 = 3\times 5$$

$$\text{LCM}(9,\ 12,\ 15) = 2^2\times 3^2\times 5$$

$$= 150 \text{ minutes}$$

The bells will toll next together after 180 minutes.

28. Solve for x : $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}, x \ne 0, \frac{2}{3}, 2.$ [3]

Ans :

We have $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}$

$$\frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$$

$$\frac{4x-3}{x(2x-3)} = \frac{1}{x-2}$$

$$(x-2)(4x-3) = 2x^2 - 3x$$

$$4x^2 - 11x + 6 = 2x^2 - 3x$$

$$2x^2 - 8x + 6 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

Thus $x = 1,3$

29. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6. [3]

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = 9$

$$a + 2d = 9 \qquad ...(1)$$

and $a_8 - a_5 = 6$

$$(a+7d) - (a+4d) = 6$$

$$3d = 6$$

$$d = 2$$

Substituting this value of d in equation (1), we get

$$a + 2(2) = 9$$

$$a = 5$$

So, A.P. is 5, 7, 9, 11, ...

or

If 7^{th} term of an A.P. is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Ans :

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9} \qquad (1)$

$$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7} \qquad (2)$$

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} = \frac{1}{63}$$

Substituting the value of d in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$

$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus $\quad a_{63} = a + (63-1)\,d$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$

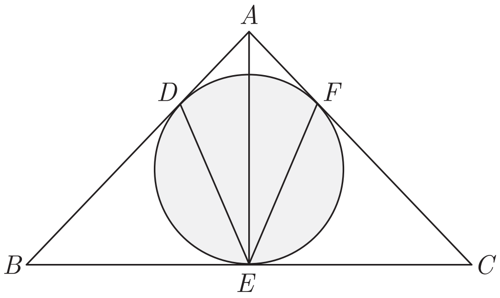
$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$

30. In $\Delta\, ABD, AB = AC$. If the interior circle of $\Delta\, ABC$ touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC. [3]

Ans :

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $\quad AF = AD \quad$ (1)

At B $\quad BE = BD \quad$ (2)

At C $\quad CE = CF \quad$ (3)

Now we have $\quad AB = AC$

$$AD + DB = AF + FC$$

$$BD = FC \qquad (AD = AF)$$

$$BE = EC \qquad (BD = BE, CE = CF)$$

Thus E bisects BC.

31. Roja, Renu and Reena are three friends. They decided to sweep a circular park near their homes. They divided the park into three parts by two equal chords AB and AC for convenience. [3]

(i) Prove that the centre of the park lies on the angle bisector of $\angle BAC$.

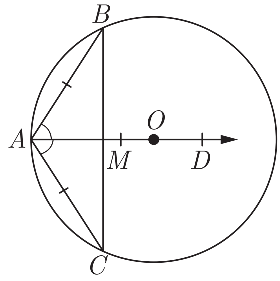
(ii) Which mathematical concept is used in the above problem?

Ans :

(i) Given : A circle $C(O, r)$ and chord $AB =$ chord AC . AD is bisector of $\angle CAB$.

To prove : Centre O lies on the bisector of $\angle BAC$.

Construction: Join BC, meeting bisector AD of $\angle BAC$, at M.



Proof : In triangles BAM and CAM,

$AB = AC$ (given)

$\angle BAM = \angle CAM$ (given)

and $\quad AM = AM$ (common)

$\Delta\, BAM \cong \Delta\, CAM$ (SAS)

$BM = CM$

and $\quad \angle BMA = \angle CMA$

As $\angle BMA + \angle CMA = 180^\circ$ (linear pair)

$\angle BMA = \angle CMA = 90^\circ$

AM is the perpendicular bisector of the chord BC.

AM passes through the centre O.

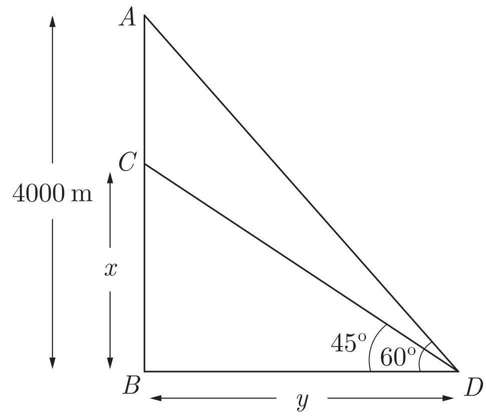
[Perpendicular bisector of chord of a circle passes through the centre of the circle]

Hence, the centre of the park lies on the angle bisector of $\angle BAC$.

32. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60^o and 45^o respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$) [3]

Ans :

Let the height first plane be $AB = 4000$ m and the height of second plane be $BC = x$ m. As per given in question we have drawn figure below.



Here $\angle BDC = 45^o$ and $\angle ADB = 60^o$

In $\Delta\, CBD$, $\quad \frac{x}{y} = \tan 45^o = 1 \Rightarrow x = y$

and in $\Delta\, ABD$, $\frac{4000}{y} = \tan 60^o = \sqrt{3}$

$$y = \frac{4000\sqrt{3}}{3}$$

$$= 2306.67 \text{ m}$$

Thus vertical distance between two,

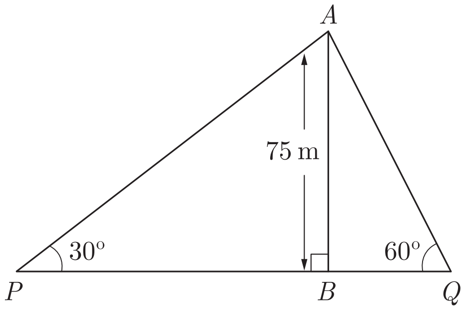
$$4000 - y = 4000 - 2306.67$$
$$= 1693.33 \text{ m}$$

or

Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. find the distance between the two men. (Use $\sqrt{3} = 1.73$)

Ans :

Let AB be the building and the two men are at P and Q. As per given in question we have drawn figure below.



In ΔABP, $\tan 30° = \frac{AB}{BP}$

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$
$$BP = 75\sqrt{3} \text{ m}$$

In ΔABQ, $\tan 60° = \frac{AB}{BQ}$

$$\sqrt{3} = \frac{75}{BQ}$$
$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

Distance between the two men,

$$PQ = BP + BQ = 75\sqrt{3} + 25\sqrt{3}$$
$$= 100\sqrt{3} = 100 \times 1.73 = 173$$

33. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use $\pi = \frac{22}{7}$ [3]

Ans :

Given,

Height of cylinder $= 2.1$ m

Radius of cylinder $=$ radius of cone $= \frac{3}{2}$ m

Slant height of cone $= 2.8$ m

Surface area of tent

$$= C.S.A \text{ of cone} + C.S.A \text{ of cylinder.}$$
$$= \pi rl + 2\pi rh = \pi r(l + 2h)$$

Area of canvas required will be surface area of tent.

Thus $\pi r(l + 2h) = \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1)$

$$= \frac{33}{7} \times 7 = 33 \text{ m}^2$$

$$\text{Total Cost} = 33 \times 500$$
$$= 16,500$$

34. A circular sheet of radius 18 centimetre is divided into 9 equal sectors. [3]

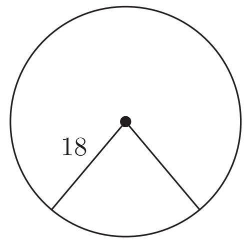
(a) Find the measure of the central angle of a sector.

(b) Find the slant height of a cone which can be made by a sector.

(c) Find the lateral surface area of the cone thus formed.

Ans :

(a)



Radius $= 18$ cm

Central angle of the circle $= 360°$

Central angle of the sector $= 40°$

(b) Slant height $= 18$ cm

$$\frac{x}{360} = \frac{r}{4}$$
$$\frac{40}{360} = \frac{r}{18}$$
$$r = 2 \text{ cm}$$

(c) Curved surface area of cone $= \pi rl$

$$\pi \times 2 \times 18 = 36\pi \text{ cm}^2$$

Section D

35. Find the other zeroes of the polynomial $x^4 - 5x^3 + 2x^2 + 10x - 8$ if it is given that two zeroes are $-\sqrt{2}$ and $\sqrt{2}$. [4]

Ans :

We have two zeroes $\sqrt{2}$ and $-\sqrt{2}$.

Two factors are $(x + \sqrt{2})$ and $(x - \sqrt{2})$

$g(x) = (x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2$ is a factor of the given polynomial

$$\begin{array}{r|l} & x^2 - 5x + 4 \\ x^2 - 2 & x^4 - 5x^3 + 2x^2 + 10x - 8 \\ & x^4 \quad\quad - 2x^2 \\ \hline & -5x^3 + 4x^2 + 10x - 8 \\ & -5x^3 \quad\quad - 10x \\ \hline & 4x^2 \quad\quad - 8 \\ & 4x^2 \quad\quad - 8 \\ \hline & 0 \end{array}$$

Quotient $= x^2 - 5x + 4 = (x - 4)(x - 1)$

Hence other zeroes are 4 and 1.

or

Find all the zeros of the polynomial $3x^4+6x^3-2x^2-10x-5$ it two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans :

$$\begin{array}{r|l} & x^2+2x+1 \\ 3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\ & 3x^4 \quad\quad -5x^2 \\ \hline & 6x^3+3x^2-10x-5 \\ & -6x^3 \quad\quad -10x \\ \hline & 3x^2 \quad\quad -5 \\ & 3x^2 \quad\quad -5 \\ \hline & 0 \end{array}$$

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of the given polynomial.

So, $\left(x-\sqrt{\frac{5}{3}}\right),\left(x+\sqrt{\frac{5}{3}}\right)$ will be its two factors

$\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2-5)$

is a factor of given polynomial

Now, dividing it by $3x^2-5$.

$$x^2+2x+1 = (x+1)^2 = (x+1)(x+1)$$

two other zeroes $= -1$ and -1

Hence all the zeroes of given polynomial

$$= \sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}, -1 \text{ and } -1$$

36. Solve the following pairs of linear equations by elimination method. [4]

(a) $x+y=5$ and $2x-3y=4$

(b) $3x+4y=10$ and $2x-2y=2$

(c) $3x-5y-4=0$ and $9x=2y+7$

Ans :

(a) We have, $x+y=5$ ...(1)

and $2x-3y=4$ ...(2)

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x+y)+(2x-3y) = 3\times 5+4$$

or, $3x+3y+2x-3y = 15+4$

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

Substituting $x=\frac{19}{5}$ in equation (1),

$$\frac{19}{5}+y = 5$$

$$y = 5-\frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y=\frac{6}{5}$

(b) We have, $3x+4y = 10$ ...(1)

and $2x-2y = 2$ ...(2)

Multiplying equation (2) by 2 and adding in (1),

$$(3x+4y)+2(2x-2y) = 10+2\times 2$$

or, $3x+4y+4x-4y = 10+4$

or, $7x = 14$

$$y = 1$$

Hence, $x=2$ and $y=1$.

(c)

We have, $3x-5y = 4$ ...(1)

and $9x = 2y+7$ ...(2)

Multiplying equation (1) by 3 and rewriting equation (2) we have

$9x-15y = 12$ ...(3)

$9x-2y = 7$ ...(4)

Subtracting equation (4) from equation (3),

$$-13y = 5$$

$$y = -\frac{5}{13}$$

Substituting value of y in equation (1),

$$3x-5\left(\frac{-5}{13}\right) = 4$$

$$3x = 4-\frac{25}{13}$$

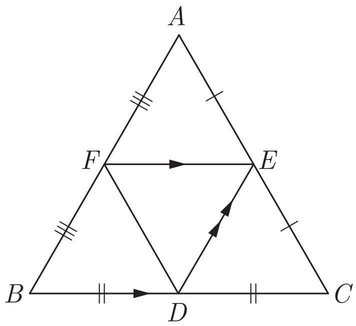
$$x = \frac{27}{13\times 3} = \frac{9}{13}$$

Hence $x = \frac{9}{13}$ and $y=-\frac{5}{13}$

37. In ΔABC, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of $ar(\Delta DEF)$ to $ar(\Delta ABC.)$ [4]

Ans :

As per given condition we have drawn the figure below. Here F, E and D are the mid-points of AB, AC and BC respectively.



Hence, $FE \| BC, DE \| AB$ and $DF \| AC$

By mid-point theorem,

If $DE \| BA$ then $DE \| BF$

and if $FE \| BC$ then $FE \| BD$

Therefore $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of Parallelogram divides it into two equal Areas.

Hence $ar(\Delta BDF) = ar(\Delta DEF)$ ...(1)

Similarly $ar(\Delta CDE) = ar(\Delta DEF)$ ...(2)

$(\Delta AFE) = ar(\Delta DEF)$ ...(3)

$(\Delta DEF) = ar(\Delta DEF)$ ...(4)

Adding equation (1), (2), (3) and (4), we have

$ar(\Delta BDF)+ar(\Delta CDE)+ar(\Delta AFE)+ar(\Delta DEF)$

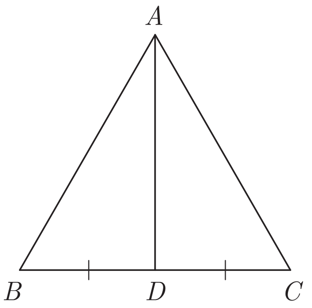
$$= 4ar(\Delta DEF)$$
$$ar(\Delta ABC) = 4ar(\Delta DEF)$$
$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

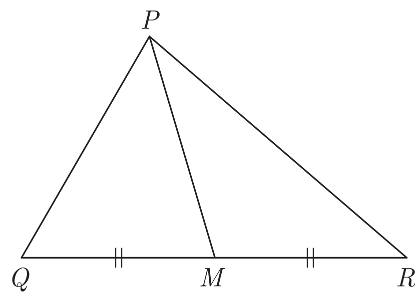
or

In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.
Prove that $\Delta ABC \sim \Delta PQR$.

Ans :

As per given condition we have drawn the figure below.





In ΔABC, AD is the median, therefore

$$BC = 2BD$$

and in ΔPQR, PM is the median,

$$QR = 2QM$$

Given, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus $\Delta ABC \sim \Delta PQR$. Hence Proved.

38. Given that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, find the values of $\tan 75^\circ$ and $\tan 90^\circ$ by taking suitable values of A and B. [4]

Ans :

We have $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

(i) $$\tan 75^\circ = \tan(45^\circ + 30^\circ)$$
$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 \cdot \tan 45^\circ \cdot \tan 30^\circ}$$
$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= \frac{3 + 2\sqrt{3} + 1}{(\sqrt{3})^2 - (1)^2} = \frac{4 + 2\sqrt{3}}{2}$$

Hence $\tan 75^\circ = 2 + \sqrt{3}$

(ii) $$\tan 90^\circ = \tan(60^\circ + 30^\circ)$$
$$= \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ}$$
$$= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$$

Hence, $\tan 90^\circ = \infty$

or

In an acute angled triangle ABC, *if* $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$, find $\angle A, \angle B$ and $\angle C$.

Ans :

We have $$\sin(A+B-C) = \frac{1}{2} = \sin 30^\circ$$

or, $$A + B - C = 30^\circ \quad ...(1)$$

and $$\cos(B+C-A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

or, $$B + C - A = 45^\circ \quad ...(2)$$

Adding equation (1) and (2), we get

$$2B = 75^\circ$$

or, $$B = 37.5^\circ$$

Now subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$

or, $$A - C = 7.5^\circ \quad ...(3)$$

Now $$A + B + C = 180^\circ$$
$$A + B + C = 180^\circ$$
$$A + C = 180^\circ - 37.5^\circ = 142.5^\circ \quad ...(4)$$

Adding equation (3) and (4), we have

$$2A = 135^\circ$$

or, $$A = 67.5^\circ$$

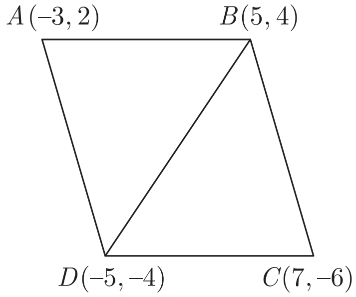
and, $$C = 75^\circ$$

Hence, $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$

39. Find the area of a quadrilateral $ABCD$, the co-ordinates of whose vertices are $A(-3,2), B(5,4), C(7,-6)$ and $D(-5,-4)$. [4]

Ans :

As per question the quadrilateral is shown below.



Area of triangle ABD

$$\Delta_{ABD} = \frac{1}{2}\left| -3(8) + 5(-6) + -5(2-4) \right|$$

$$= 22 \text{ sq. units}$$

Area of triangle BCD

$$\Delta_{BCD} = \frac{1}{2}\left| 5(-2) + 7(-8) - 5(10) \right|$$

$$= 58 \text{ sq. units}$$

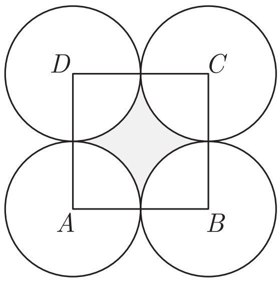
$$\text{Area}_{ABCD} = \Delta_{ABD} + \Delta_{BCD}$$

$$= 22 + 58 = 80 \text{ sq. units}$$

40. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circle is $\frac{24}{7}$ cm2. Find the radius of each circle. [4]

Ans :

As per question statement the figure is shown below.



Let r cm be the radius of each circle.

Area of square – Area of 4 sectors $= \frac{24}{7}$ cm^2

$$(2r)^2 - 4\left(\frac{90}{360^{\circ}} \times \pi r^2\right) = \frac{24}{7}$$

$$4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$$

$$\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

$$6r^2 = 24$$

$$r^2 = 4$$

$$r = \pm 2$$

Thus radius of each circle is 2 cm.